

Reply to “Comment on ‘Two Fermi points in the exact solution of the Kondo problem’”

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Abstract

In his Comment N. Andrei questions the use of symmetric limits of integration of the Bethe ansatz integral equations in my recent study devoted to some features of the Bethe ansatz solution of the Kondo problem. In this Reply I show that the statement of the Comment about the asymmetry of integration limits contradicts the distribution of the quantum numbers in discrete Bethe ansatz equations. I also argue that the asymmetry of the excitation energy, supported in the Comment, contradicts the initial chiral symmetry of conduction electrons in the physical Kondo problem. Hence the distribution of spin rapidities has to be symmetric for any magnetization.

The Comment by N. Andrei [1] is devoted to the discussion of the very important point of the Kondo problem. Recently I pointed out that two energy scales can appear in the Bethe ansatz solution of the Kondo problem [2]. The onset of the second scale was the consequence of the symmetric with respect to zero distribution of spin rapidities, which parametrize the eigenvalues and eigenstates of the Hamiltonian of the Kondo problem in the Bethe ansatz approach. The main question raised by the Comment is whether that distribution of spin rapidities is symmetric or asymmetric. In other words, the Comment questions the use of symmetric limits of integration of the Bethe ansatz integral equations in Ref. [2]. I note that for the presence of two energy scales in the Bethe ansatz solution it is enough to consider both of limits being not necessary symmetric, but non-infinite, though. One infinite and one finite limits of integration produce ill-defined features of the ground states of the impurity models in the Bethe ansatz solutions and it was necessary to introduce special artificial cut-off procedures to avoid these problems [3].

There are two arguments that support the symmetric limits of integration of my approach:

1. In the discrete Bethe ansatz equations for spin rapidities [Eqs. (1) of the Comment or Eqs. (3) of Ref. [2]] the quantum numbers J_α , which parametrize the eigenstates and eigenvalues are distributed *symmetrically* with respect to zero, between $\pm(N - M - 1)/2$ [1, 2]. Here N is the number of electrons and M is the number of electrons with down spin (including the impurity). Hence, this symmetric distribution is valid for *any* M , i.e., for any magnetic moment of the system [the z -projection of the total spin of the system is equal to $S^z = (N/2) - M$]. The integral Bethe ansatz equations (Eq. (4) of the Comment and Eq. (4) of Ref. [2]) are the direct consequences of discrete Bethe ansatz equations (Eqs. (1) of the Comment or Eqs. (3) of Ref. [2]), according to the well-known procedure [4]. Hence the distribution of rapidities in the thermodynamic continuum limit must be also symmetric for any magnetization of the system, otherwise the distributions of quantum numbers in the discrete Bethe ansatz equations and in their continuum limit would contradict each other.

2. The energy of any excitation, mentioned in the Comment, is, naturally, the consequence of the Bethe ansatz equations for quantum numbers (or for the distribution of rapidities). One cannot obtain that energy independently of the Bethe ansatz equations. N. Andrei points out that this energy is the asymmetric function of spin rapidities in

the known Bethe ansatz solution of the Kondo problem. I argue that this is the artifact of the approach. Physically, one can consider both signs for the kinetic term in the Hamiltonian of the problem (the first equation of the Comment and Eq. (1) of Ref. [2]) because one can study the Kondo interaction of the magnetic impurity with either right- or left-moving conduction electrons from the same grounds. The physical answer must not depend on the choice of the chirality of conduction electrons, naturally. However, if one uses the definition of the energy supported in the Comment, the energy changes its sign when changing the chirality of conduction electrons, namely due to that asymmetry. Clearly, one has to use the symmetrized energy, too, to avoid this inconsistency. That implies that the energy should be defined as $E = -\sum_{j=1}^N |k_j| + \text{const}$ [5]. This removes the inconsistency between the symmetric distribution of quantum numbers in the discrete Bethe ansatz equations and the asymmetric distribution in its continuous limit [Eq. (4) of the Comment], mentioned above. It turns out that, emphasized in the Comment, distributions of rapidities and energies are symmetric functions with respect to zero in any known Bethe ansatz solutions (lattice ones or quantum field theories) *except of impurity problems*.

Summarizing, the statement of the Comment about the asymmetry of integration limits contradicts the distribution of the quantum numbers in discrete Bethe ansatz equations. On the other hand, the asymmetry of the excitation energy supported in the Comment, contradicts the initial chiral symmetry of conduction electrons in the physical Kondo problem. Therefore the distribution of spin rapidities has to be symmetric for any magnetization of the system, which yields the presence of the second energy scale in the Bethe ansatz solution of the Kondo problem. This confirms the correctness of the conclusions made in my study [2].

References

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